Basics of Spatial filtering :-

* Subimage operation :-
* filter, mask, kernel, template (or) window.

↓

Values in a filter image called Co-efficients.

* Concept of filtering has its roots in the use of F T for signal processing ⇒ Frequency domain.

Spatial filtering

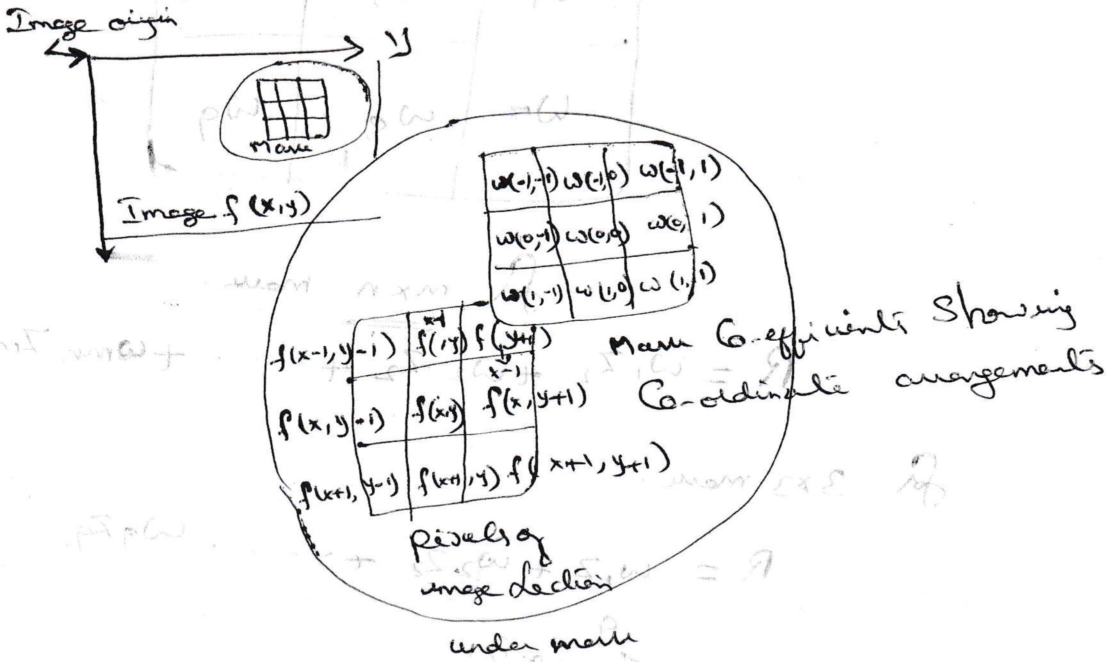

Fig : Mechanics of Spatial filtering

Result (Response) ⇒ R of linear filtering with the filter mask at a point (x,y)

$$R = \omega(-1,-1)\, f(x-1,y-1) + \omega(-1,0)\, f(x-1,y) + \dots$$
$$\omega(0,0)\, f(x,y) + \dots + \omega(1,0)\, f(x+1,y)$$
$$+ \omega(1,1)\, f(x+1,y+1).$$

(iii)

In general linear filtering of an image f of size M×N with a filter mask of size m×n is given by

$$\boxed{g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)} \quad \text{— (1)}$$

Representation of 3×3 spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

for m×n mask.

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_{mn} z_{mn}.$$

for 3×3 mask.

$$R = w_1 z_1 + w_2 z_2 + \cdots \cdots w_9 z_9.$$

$$= \sum_{i=1}^{9} w_i z_i.$$

Smoothing Spatial filters

* used for blurring & for noise reduction.
* Blurring → Removal of small details from an image prior to object extraction & bridging of small gaps in lines or curves.

Noise reduction ⟹ Blurring with a linear filter & also by non linear filtering.

i) Smoothing linear filters
ii) order statistics filters.

Smoothing linear filters.

* o/p (response) ⟹ value of each pixel replaced by Average of pixels contained in the neighborhood of the filter mask.

⟹ averaging filters.

⟹ Low pass filters.

* Replacing value of every pixel in an image by the average of the grey levels in the neighborhood defined by the filter mask. ⟹ reduced sharp transitions in grey levels.

random noise has

averaging ⟹ { blur edges. → characterised by sharp transitions in gray levels.

↳ side effect of averaging.

⟹ reduction of irrelevant details in an image

$$R = 1/9 \sum_{i=1}^{9} z_i.$$

$$\underbrace{\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{box filter}} \qquad \underbrace{\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}}_{\text{weighted average}}$$

[all Co-efficients are equal]

⇓

blurring in smoothing process.

Application of Spatial averaging ⇒ blur an image / Intensity of Small objects blends with the background & larger objects become blob like & easy to

Order - Statistics filter [non linear Spatial filters]

Response ⇒ ordering (ranking) the pixels in the image area [encompassed] by filter & replacing the value of the center pixel with the value determined by ranking result.

⇒ Median filter :

* [replaces value of a pixel by median of grey levels in the neighborhood of that pixel]

* Less blurring than linear Smoothing filters.

* Effective in presence of impulse noise & Salt & pepper noise [app white & black dots].

* (10, 20, 20, 20, 15, 20, 20, 25, 100) => (10, 15, 20, 20, 20, 20, 20, 25, 100).
i.e median is 20.

* 100th percentile => max filter => brightest points.
50th " => Min filter => darker ".

Sharpening Spatial filters.

* Sharpening => highlight fine detail in an image [enhance detail that has been blurred].
⇓
Image Acquisition

* Integration => Averaging, differentiation => opposite of Averaging

First derivative

1) Must be zero in flat segments (constant gray level Values).
2) " " nonzero at onset of a gray level step (or) ramp
3) " " " along ramps.

Second derivative

1) must be zero in flat areas.
2) " " nonzero at the onset and end of a gray level step (or) ramp.
3) must be zero along ramps of a constant slope.

Definition

First order derivative of a one-dimensional function $f(x)$ is the difference.

$$\frac{\partial f}{\partial x} = f(x+1) - f(x).$$

Second order derivative as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

First order derivative

* It generally produce thicker edges in an image.
* It has a stronger response to a grey level step.

Second order derivative

* It has a stronger response to fine detail such as thin lines and isolated points.
* It produces a double response at step ~~at step~~ changes in grey level.

IInd derivative is better suited than the first derivative for Image enchancement.

Use of Second derivatives for Enhancement - Laplacian

Isotropic filters ⇒ response is independent of the direction of the discontinuities in the image.

↓ rotation Invariant.

Simplest Isotropic filter ↑ Laplacian. [Linear operator ⇒ deri of any order].

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

$$\frac{\partial^2 f}{\partial^2 x^2} = f(x+1, y) + f(x-1, y) - 2 f(x, y) \quad \text{— (1)}$$

& $$\frac{\partial^2 f}{\partial^2 y^2} = f(x, y+1) + f(x, y-1) - 2 f(x, y) \quad \text{— (2).}$$

$$\therefore \nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) \cancel{-4} - 4 f(x, y) \quad \text{— (3)}$$

$$g(x,y) = \begin{cases} A f(x,y) - \nabla^2 f(x,y) & \rightarrow \text{if Center coeffi of the Lap mask is -ve.} \\ A f(x,y) + \nabla^2 f(x,y) & \rightarrow \text{If Center " of the " " " +ve.} \end{cases}$$

f_{hb} =

* Background tonality was perfectly preserved.

Composite Laplacian

0	-1	0
-1	-4	-1
0	-1	0

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A+4

Filter mask to imp Laplacian

A+8

Second composite mask.

-1	-1	-1
-1	-8	-1
-1	-1	-1

9

Mask used to implement an extension of this eqn includes diagonal neighbours.

High boost filtered Image

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

2 other implementations of Laplacian.

unsharp masking & high boost filtering :

Publishing industry [Sharpen => subtracting blur from an image]

=> unsharp masking. $\boxed{f_s(x,y) = f(x,y) - \bar{f}(x,y)}$ —— (1)

where $f_s(x,y)$ => Sharpend image obtained by unsharp masking

$f(x,y)$ => original image.

$\bar{f}(x,y)$ => blurred version of $f(x,y)$.

Ex: dark room photography.

* Generalization of unsharp masking => high boost filtering

High boost filtered image,

$$\boxed{f_{hb}(x,y) = A f(x,y) - \bar{f}(x,y)}$$ (2) where $A \geq 1$

rewritten [Σ>

$$f_{hb}(x,y) = (A-1) f(x,y) + f(x,y) - \bar{f}(x,y) \quad \text{—— (3)}$$

$$= A f(x,y) - f(x,y) - \bar{f}(x,y) + f(x,y)$$

Sub (1) in (3) => $f_{hb}(x,y) = A f(x,y) + f_s(x,y)$ —— (4)

use of first derivatives for enhancement – The Gradient

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} \quad \text{——— (1)}$$

Magnitude

$$\nabla f = \text{mag}\,(\nabla f)$$

$$= \left[G_x^2 + G_y^2 \right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \right]^{1/2} \quad \text{——— (2)}$$

* partial derivatives of (1) are not rotation invariant, but mag is gradient

* Computation burden => It is not trivial.

hence app this by absolute values.

$$\nabla f \approx |G_x| + |G_y|. \quad \text{——— (4)}$$

$$G_x = (z_9 - z_5) \;\&\; G_y = (z_8 - z_6). \quad \text{——— (5)}$$

$$\therefore \nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2 \right]^{1/2} \quad \text{——— (6)}$$

$$\therefore \nabla f \approx |z_9 - z_5| + |z_8 - z_6| \quad \text{——— (7)}$$

Robert cross-gradient operators.

* Many of even size are awkward to implement

$$\nabla f = |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|.$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Rightarrow \text{Robert.}$$

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow \text{Sobel.}$$

* Median filtering => non linear process of removing image features.
  => unacceptable in medical imaging.
* Alternate is gradient [compared to laplacian].
* Enhancing => higher content of visual detail.

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt operators