SLANT Transform:-

* $N \times N$ Slant transform matrices are defined by the recursion:

$$S_n = \frac{1}{\sqrt{2}} \left[\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ a_n & b_n & & -a_n & b_n & \\ \hline 0 & & I_{(N/2)-2} & 0 & & I_{(N/2)-2} \\ \hline 0 & 1 & 0 & 0 & -1 & 0 \\ -b_n & a_n & & b_n & a_n & \\ \hline 0 & & I_{(N/2)-2} & 0 & & -I_{(N/2)-2} \end{array}\right] \left[\begin{array}{c|c} S_{n-1} & 0 \\ \hline 0 & S_{n-1} \end{array}\right]$$

where $N = 2^n$, I_M denotes an $M \times M$ Identity matrix

and

$$\boxed{S_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}} \quad (1)$$

* The parameters a_n & b_n are defined by the recursions.

$$b_n = (1 + 4a_n^2 - 1)^{-1/2} \qquad a_1 = 1$$

$$a_n = 2b_n a_{n-1}$$

which solve's to give

$$a_{n+1} = \left(\frac{3N^2}{4N^2 - 1}\right)^{1/2}$$

$$b_{n+1} = \left(\frac{N^2 - 1}{4N^2 - 1}\right)^{1/2}, \quad N = 2^n.$$

* using these formulas 4×4 Slant transformation is obtained as:

$$S_2 = \frac{1}{2}\begin{array}{c} \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ \frac{3}{\sqrt5} & \frac{1}{\sqrt5} & \frac{-1}{\sqrt5} & \frac{-3}{\sqrt5} \\ 1 & -1 & -1 & 1 \\ \frac{1}{\sqrt5} & \frac{-3}{\sqrt5} & \frac{3}{\sqrt5} & \frac{-1}{\sqrt5} \end{array}\right] \end{array} \begin{array}{c} \text{Sequency} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

(X) refer back for derivation

Properties Of Slant transform.

* The Slant transform is real & orthogonal.

$$\left.\begin{array}{l} \therefore \boxed{S = S^*} \\ \boxed{S^{-1} = S^T} \end{array}\right\} \text{—— (i)}$$

* The Slant transform is a fast transform which can be implemented in $O(N \log_2 N)$ operations on an $N \times 1$ vector.

* It has very good to excellent energy compaction for images.

* The basis vectors of the slant transform matrix (S are not sequency ordered for $n \geq 3$, If S_{n-1} is sequency ordered, the ith row sequency of S_n is given as follows.

$i = 0$ Sequency $= 0$

$i = 1$ Sequency $= 1$

$2 \leq i \leq N/2 - 1$ Sequency $\begin{cases} 2i & i = \text{even} \\ 2i+1 & i = \text{odd} \end{cases}$

$i = N/2$ Sequency $= 2$.

$i = N/2 + 1$ Sequency $= 3$.

$N/2 + 2 \leq i \leq N - 1$ Sequency $= \begin{cases} 2(i - N/2) + 1 & i = \text{even} \\ 2(i - N/2), & i = \text{odd} \end{cases}$

Derivation for $N = 4 \implies 4\times 4$ Slant transformation is given by.

$$S_2 = \frac{1}{\sqrt{2}}\left[\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ a_2 & b_2 & & -a_2 & b_2 & \\ \hline \multicolumn{2}{c|}{0} & I_{(4/2)-2} & \multicolumn{2}{c|}{0} & I_{(4/2)-2} \\ \hline 0 & 1 & 0 & 0 & -1 & 0 \\ -b_2 & a_2 & & b_2 & a_2 & \\ \hline \multicolumn{2}{c|}{0} & I_{(4/2)-2} & \multicolumn{2}{c|}{0} & -I_{(4/2)-2} \end{array}\right]\left[\begin{array}{c|c} S_1 & 0 \\ \hline 0 & S_1 \end{array}\right]$$

W.K.T.

$$a_{n+1} = \left(\frac{3N^2}{4N^2-1}\right)^{1/2}, \quad b_{n+1} = \left(\frac{N^2-1}{4N^2-1}\right)^{1/2}, \quad N = 2^n.$$

we have to calculate a_2 & b_2 hence $n=1, \implies N=2$.

$$\therefore a_{1+1} = \left(\frac{3\times 4}{4\times 4-1}\right)^{1/2}, \quad b_{1+1} = \left(\frac{4-1}{4\times 4-1}\right)^{1/2}.$$

$$a_2 = \left(\frac{\cancel{3}\times 4}{\cancel{3}\times 5}\right)^{1/2}, \quad b_2 = \left(\frac{\cancel{3}}{\cancel{3}\times 5}\right)^{1/2}.$$

$$\therefore \boxed{a_2 = 2/\sqrt{5}} \quad \boxed{b_2 = 1/\sqrt{5}}$$

$$\implies S_2 = \frac{1}{\sqrt{2}}\left[\begin{array}{cc|c|cc|c} 1 & 0 & 0 & 1 & 0 & 0 \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & \\ \hline \multicolumn{2}{c|}{0} & I_0 & \multicolumn{2}{c|}{0} & I_0 \\ \hline 0 & 1 & 0 & 0 & -1 & 0 \\ -\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & \\ \hline \multicolumn{2}{c|}{0} & I_0 & \multicolumn{2}{c|}{0} & -I_0 \end{array}\right]\left[\begin{array}{c|c} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & 0 \\ \hline 0 & S_1 \end{array}\right]$$

$$\therefore\ S_2 = \frac{1}{\sqrt{2}}\left[\begin{array}{c|c} \begin{pmatrix}1 & 0\\ 2/\sqrt5 & 1/\sqrt5\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} & \begin{pmatrix}1 & 0\\ -2/\sqrt5 & 1/\sqrt5\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \\ \hline \begin{pmatrix}0 & 1\\ -1/\sqrt5 & 2/\sqrt5\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} & \begin{pmatrix}0 & -1\\ 1/\sqrt5 & -2/\sqrt5\end{pmatrix}\begin{pmatrix}+1 & 0\\ 0 & +1\end{pmatrix}\end{array}\right]\left[\begin{array}{c|c} S_1 & 0\\ \hline 0 & S_1\end{array}\right]$$

Not necessary

$$\therefore\ S_2 = \frac{1}{\sqrt{2}}\left[\begin{array}{c|c} \begin{pmatrix}1+0 & 0+0\\ 2/\sqrt5+0 & 0+1/\sqrt5\end{pmatrix} & \begin{pmatrix}1+0 & 0+0\\ -2/\sqrt5+0 & 0+1/\sqrt5\end{pmatrix} \\ \hline \begin{pmatrix}0+0 & 0+1\\ -1/\sqrt5+0 & 0+2/\sqrt5\end{pmatrix} & \begin{pmatrix}0+0 & 0-1\\ +1/\sqrt5+0 & 0+(+2/\sqrt5)\end{pmatrix}\end{array}\right]\left[\begin{array}{c|c} S_1 & 0\\ \hline 0 & S_1\end{array}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\begin{array}{c|c} \begin{pmatrix}1 & 0\\ 2/\sqrt5 & 1/\sqrt5\end{pmatrix} & \begin{pmatrix}1 & 0\\ -2/\sqrt5 & 1/\sqrt5\end{pmatrix} \\ \hline \begin{pmatrix}0 & 1\\ -1/\sqrt5 & 2/\sqrt5\end{pmatrix} & \begin{pmatrix}0 & -1\\ +1/\sqrt5 & +2/\sqrt5\end{pmatrix}\end{array}\right]\left[\begin{array}{c|c} S_1 & 0\\ \hline 0 & S_1\end{array}\right]$$

$$= 1/\sqrt2\left[\begin{array}{cc} \begin{pmatrix}1 & 0\\ 2/\sqrt5 & 1/\sqrt5\end{pmatrix} 1/\sqrt2\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & \begin{pmatrix}1 & 0\\ -2/\sqrt5 & 1/\sqrt5\end{pmatrix} 1/\sqrt2\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \\ \begin{pmatrix}0 & 1\\ -1/\sqrt5 & 2/\sqrt5\end{pmatrix} 1/\sqrt2\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & \begin{pmatrix}0 & -1\\ +1/\sqrt5 & +2/\sqrt5\end{pmatrix} 1/\sqrt2\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\end{array}\right]$$

$$= \frac{1}{\sqrt2}\cdot\frac{1}{\sqrt2}\begin{bmatrix} 1+0 & 1+0 & 1+0 & 1+0\\ 2/\sqrt5+1/\sqrt5 & 2/\sqrt5-1/\sqrt5 & -2/\sqrt5+1/\sqrt5 & -2/\sqrt5-1/\sqrt5\\ 0+1 & 0-1 & 0-1 & 0+1\\ -1/\sqrt5+2/\sqrt5 & -1/\sqrt5-2/\sqrt5 & +1/\sqrt5+2/\sqrt5 & 1/\sqrt5-2/\sqrt5 \end{bmatrix}$$

$$\therefore S_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & 1/\sqrt{5} & -1/\sqrt{5} & -\frac{3}{\sqrt{5}} \\ 1 & -1 & -1 & 1 \\ \frac{1}{\sqrt{5}} & \frac{-3}{\sqrt{5}} & \frac{3}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{bmatrix}$$