Digital Image Processing

Introduction

* One Picture is more Worth than 10 K words.

Applns
- → Improvement of pictorial Information for human Interpretation.
- → Processing of image data
  - → Storage
  - → T_x^{on}
  - → Rep of autonomous machine Percep

* DIP

* Image --defined→ 2D f(x,y), where x & y are spatial Co-ordinates
  → Amp of f at any pair of Co-ordinates (x,y) ⇒ Intensity (or) gray level of the image at that pt.

* when x & y & Intensity values of f are all finite, discrete qty's ⇒ Digital image

* Processing of Digital images by means of a digital Computer

* It is Composed of finite no. of elements ⇒ has a particular location & value ⇒ picture elements, image elements, pels, pixels.

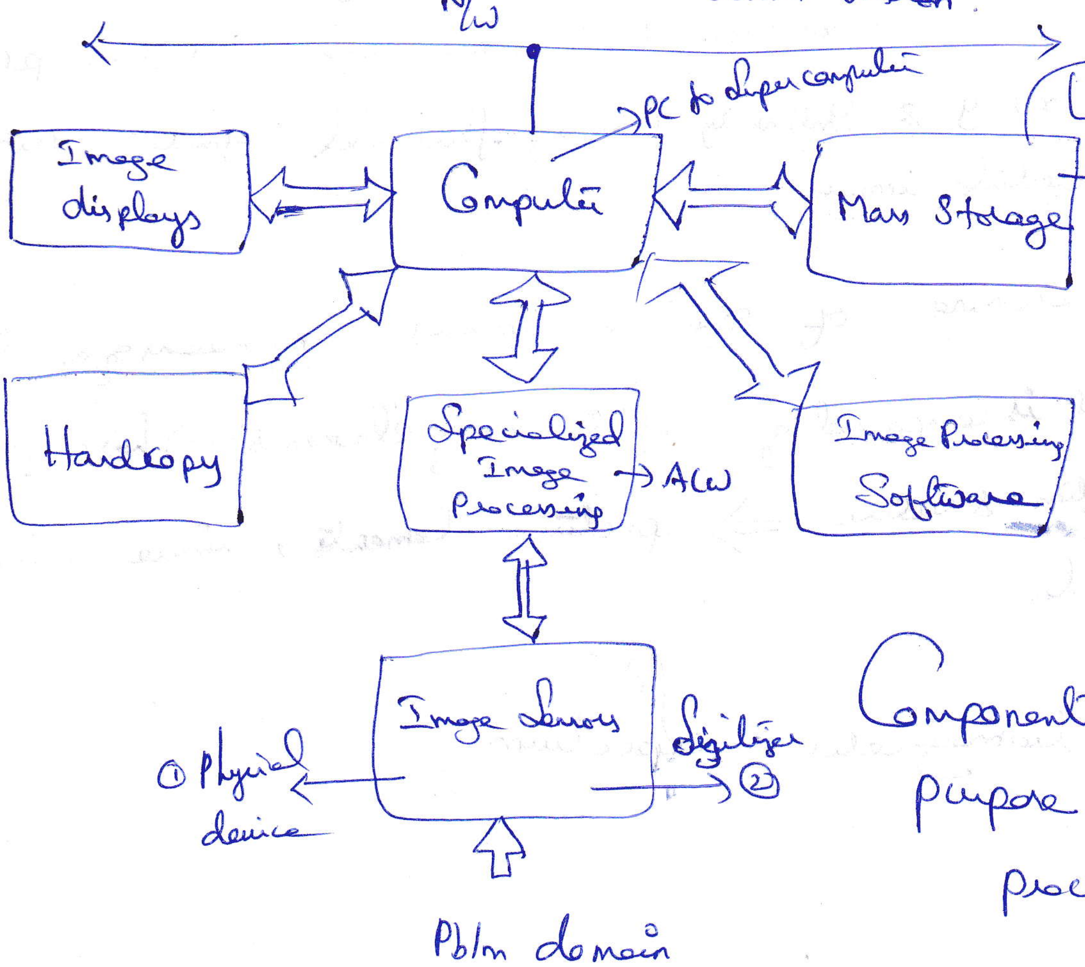
⇒ Electromagnetic Spectrum

i] Low level processing → Image Preprocessing → reduce noise
contrast Enhancem[ent]
& Image Sharpe[ning]

ii] Mid-level processing → Segmentation {partitioning an image into regions (or) objects} → I/p & o/p image[s]
→ Description of those objects } reduce
→ Classification (recognition) of Indi[vidual] object.

⇑ I/p's are images
o/p's are attributes
[edges, contours & Identity of individual objects]

iii] Higher level processing ⇒ Making sense of an ensemble of recognized objects, as in Image Analysis ⇒ associated with Vision.



frame buffers (
→ zoom, scroll & pan
→ 1024 X 1024 p[els]
each pel with 8
⇓
requires
1MB St[orage]
if image [is]
not compre[ssed]

Components of a General purpose Image processing System

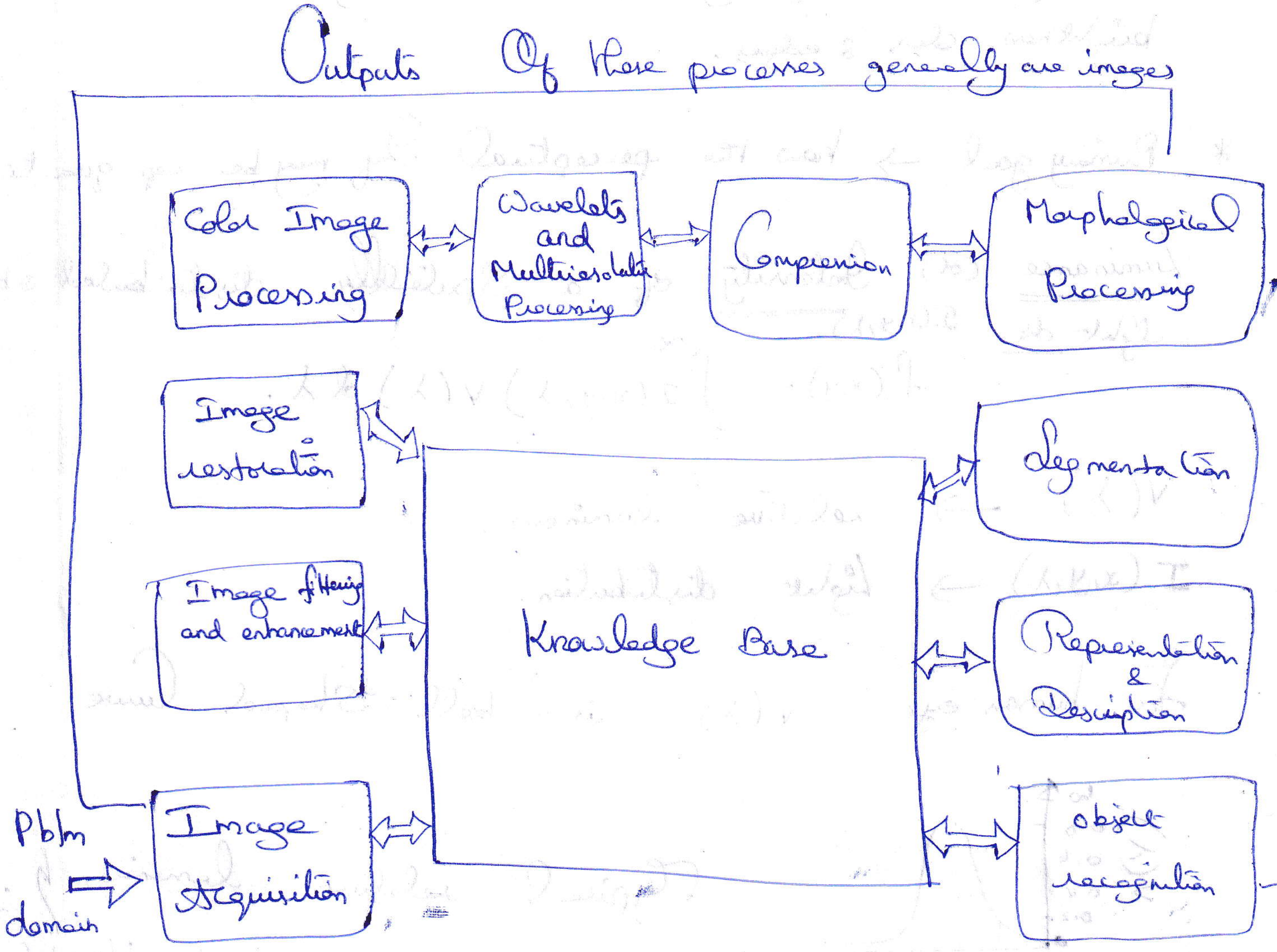


Fig ①: Fundamental Steps in Digital Image Processing

* Segmentation
  - → rugged ⇒ Successful soln
  - → Erratic ⇒ Eventual failure

* Representation
  - → Boundary rep ⇒ External Shape characteristics such as Corners & inflections
  - → Regional rep ⇒ Internal properties such as texture (or) skeletal shape

* The perceived Img may be represented by attributes such as brightness, color & edges.

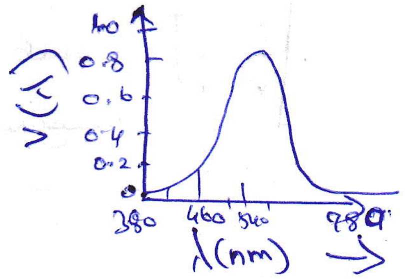
* Primary goal → how the perceptual Img may be rep quantitative

Luminance (or) Intensity of a Spatially distributed object
light dis $I(x,y,\lambda)$

$$f(x,y) = \int_0^\infty I(x,y,\lambda)\, V(\lambda)\, d\lambda.$$

$V(\lambda)$ → relative luminous.
$I(x,y,\lambda)$ → Light distribution.

For human eye, $V(\lambda)$ is a bell Shaped Curve,



Typical relative luminous of func

350 nm to 780 nm ⇒ Visible region

$V(\lambda)$ ⇒ whose characteristics depend on whether it is scotopic (or) Photopic vision.

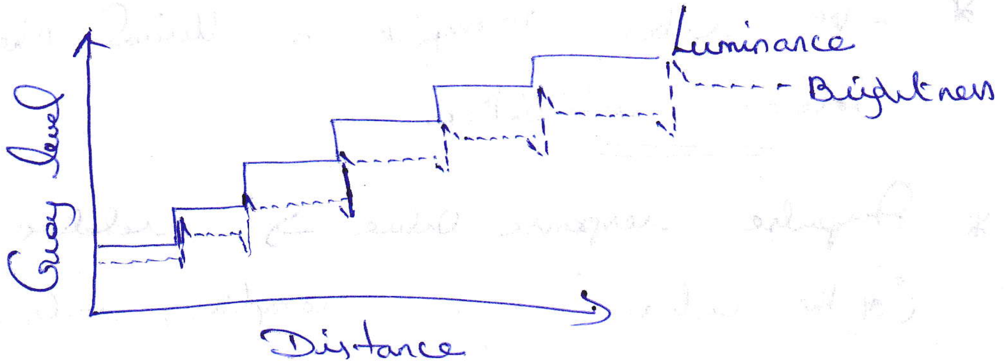
* Luminance of an Object is Independent of the luminance of the surrounding objects.

* The brightness [apparent brightness] of an object is the perceived luminance and depends on the luminance of the surround.

* 2 objects with diff illum could have identical luminances but diff brightnesses.

Mach bands

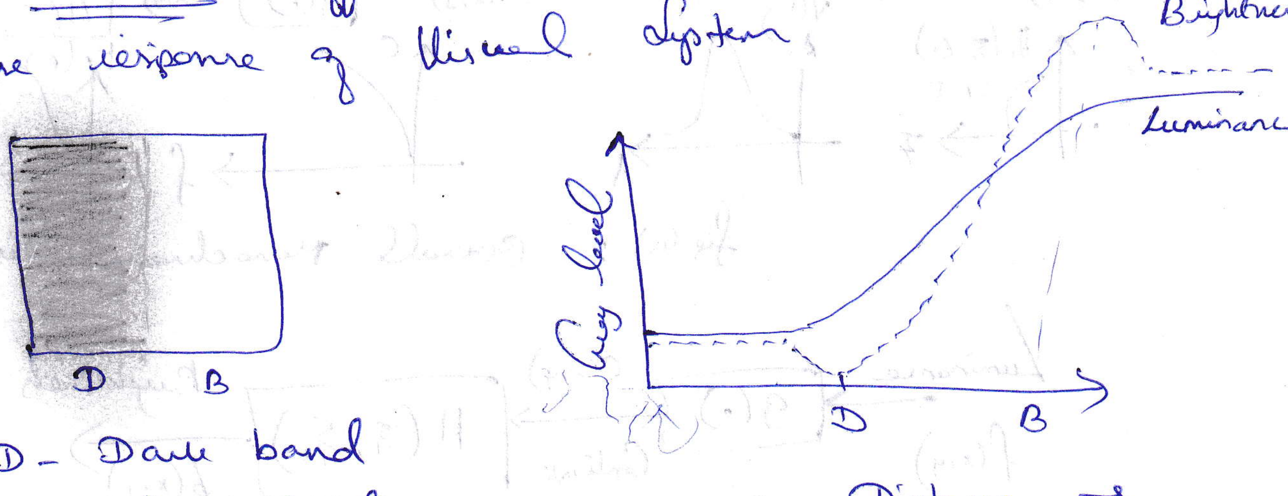
Spatial interaction of luminances from an object & its surround creates a phenomenon called the Mach band effect.



ⓐ Luminance Vs brightness

Fig ①: Mach band effect.

* It measures the response of visual system in spatial coordinates. Mach band effect can be used to estimate the impulse response of visual system.



D – Dark band
B – Bright band

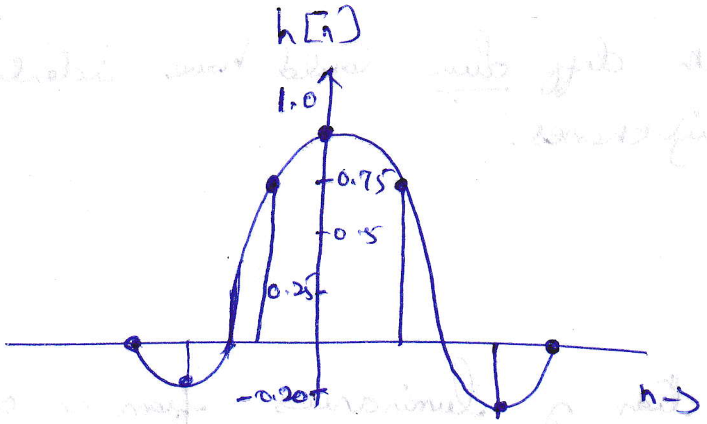


fig (2) : Nature of visual system impulse response

* −ve lobes manifest a visual phenomenon known as lateral inhibition.

* Impulse response value → relative spatial weighting (of the contrast) by the receptors, rods & cones.

* −ve lobes indicate that the neural (post retinal) signal at a given location has been inhibited by some of the laterally located receptors.

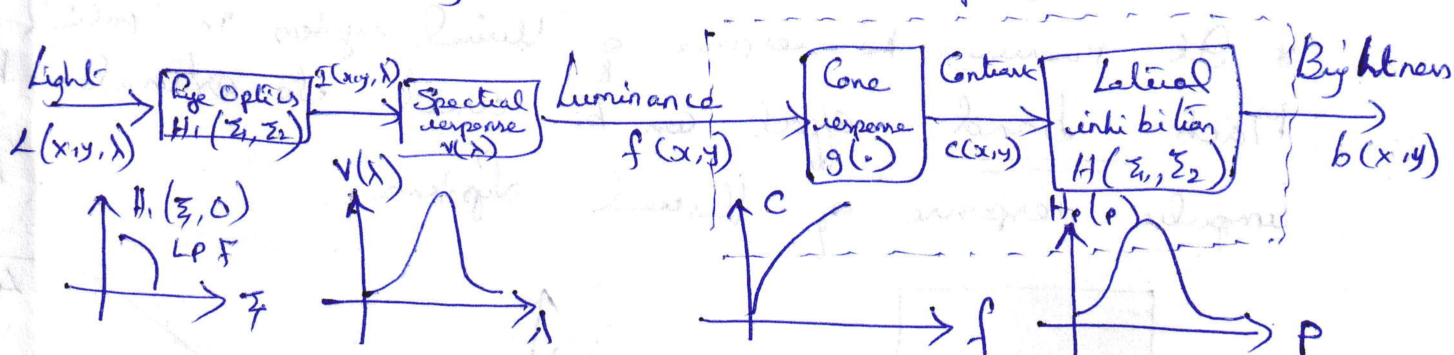


fig (a) : Overall monochrome vision model.

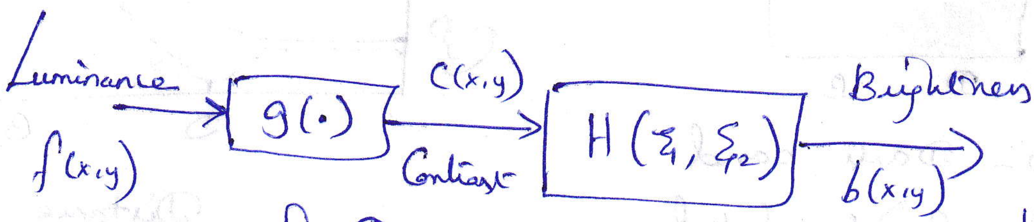


fig (b) : Simplified monochrome vision model.

A Simple Image Formation Model

$$0 < f(x,y) < \infty \quad \text{--- (1)}$$

$$f(x,y) = i(x,y)\, r(x,y) \quad \text{--- (2)}$$

where

$$0 < i(x,y) < \infty \quad \text{--- (3)}$$

$$0 < r(x,y) < 1 \quad \text{--- (4)}$$

0 → total absorption; 1 → total reflectance

* Intensity of a monochrome image at any Coordinates (x_0, y_0) be denoted by.

$$l = f(x_0, y_0)$$

$$L_{min} \leq l \leq L_{max}$$

L_{min} → +ve; L_{max} → finite ⟹ Theory.

$L_{min} = i_{min} r_{min}$ $\quad$ $L_{max} = i_{max} \cdot r_{max}$ ⟹ Practise.

* Interval $[L_{min}, L_{max}]$ called gray (or Intensity Scale.
* Common Practice ⟹ $[0, L-1]$ where $l=0$ is considered to be black & $l = L-1$ is considered to be white.
* All Intermediate Values are shades of gray varying from black to white.

Image Sampling and Quantization

Digital image

Continuous sensed data $\Longrightarrow$ digital form.
(→ Sampling → Quantization ↓)

Digitizing Co-ordinate Values → Sampling
Digitizing Amplitude " → Quantization.

Representing Digital Images.

$$f[x,y] = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & & \vdots \\ \vdots & \vdots & & \\ f(M-1,0) & f(M-1,1) & & f(M-1,N-1) \end{bmatrix}$$

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & & a_{0,N-1} \\ a_{1,0} & \vdots & & \vdots \\ \vdots & & & \\ a_{M-1,0} & a_{M-1,1} & & a_{M-1,N-1} \end{bmatrix}$$

* Digitization Process → Values of M, N & L.

$$\boxed{L = 2^k}$$

* Discrete levels are equally spaced & they are integers in the Interval $[0, L-1]$.
* Range of values spanned by the gray scale ⇒ Dynamic range

Dynamic range of an Imaging system ⟹ def to be the ratio of maximum measurable Intensity to the minimum detectable Intensity level in the system.

Rule

upper limit → Saturation

Lower limit → noise.

Contrast

Difference in Intensity between the highest & lowest Intensity levels in an image.

↑ Dynamic range ⟹ ↑ Contrast

↓ " " ⟹ dull washed out gray look.

* No of bits to be Stored. $b = M \times N \times k$
* when $M=N$, this eqn becomes $b = N^2 k$.
* Note:

Storage requirements for 8 bit images of size 1024 x 1024 & higher are not significant.

Spatial and Intensity Resolution

* Spatial resolution is a measure of the smallest discernible detail in an image.

Intensity resolution → smallest discernible change in Intensity level.

Image Interpolation

It is a basic tool used extensively in tasks such as zooming, shrinking, rotating and geometric corrections.

Some basic Relationships between Pixels

i) Neighbors of a pixel

* Pel P has four horizontal & Vertical neighbors, whose co-ordinates are given by

$(x+1, y)$, $(x-1, y)$, $(x, y+1)$ & $(x, y-1)$.

⟹ 4 Neighbors of P $N_4(P)$.
each pel is a unit distance from (x,y).

* Diagonal neighbors $[N_D(P)]$.

The four diagonal neighbors of p have Co-ordinates

$(x+1, y+1)$, $(x+1, y-1)$, $(x-1, y+1)$ & $(x-1, y-1)$ ⟹ $N_D(P)$

These pts together with 4 neighbours are called 8 neighbours of a pixel. $N_8(P)$.

Toeplitz Matrix

A Toeplitz matrix T is a matrix that has constant elements along the main diagonal and the subdiagonals. This means the elements $t(m,n)$ depend only on the difference $m-n$, i.e $t(m,n) = t_{m-n}$. It describes I/O transformation of 1D Linear shift invariant systems & correlation matrices of stationary sequences.

$$T = \begin{bmatrix} t_0 & t_{-1}, & \cdots & t_{-N+1} \\ t_1 & t_0 & t_{-1} & t_{-N+2} \\ t_2 & & & \vdots \\ \vdots & & & t_{-1} \\ t_{N-1} & \cdots\, t_2 & t_1 & t_0 \end{bmatrix}$$

Circulant Matrix

A matrix C is circulant if each of its rows (or columns) is a circular shift of the previous row (or column), i.e

$$C = \begin{bmatrix} c_0 & c_1 & c_2 \cdots & c_{N-1} \\ c_{N-1} & c_0 & c_1 & c_{N-2} \\ \vdots & & & \vdots \\ c_2 & & & c_1 \\ c_1 & c_2 \cdots & c_{N-1} & c_0 \end{bmatrix}$$

Note that C is also Toeplitz and

$$c(m,n) = c\big((m-n) \text{ modulo } N\big)$$

It describes the I/O behaviour of 1D linear periodic systems and correlation matrices of periodic sequences.

Orthogonal and unitary matrices

An orthogonal matrix is such that its Inverse equal to its transpose, i.e. A is orthogonal if

$$A^{-1} = A^T$$

$$A^T A = A A^T = I$$

A Matrix is called unitary if its Inverse is equal to its Conjugate transpose.

$$A^{-1} = \overset{*}{A}{}^T$$

$$A A^{*T} = A^{*T} A = I.$$

A real orthogonal matrix is also unitary, but a unitary matrix need not be orthogonal.

Block Matrices

Any Matrix A whose elements are matrices themselves is called a block matrix.

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,n} \\ A_{2,1} & A_{2,2} & & A_{2,n} \\ \vdots & & & \\ A_{m,1} & A_{m,2} & \cdots & A_{m,n} \end{bmatrix}$$